Origins of a 14th Century Turin Shroud Image

Joseph S. Accetta

A substantial part of this paper was written while the author was at Georgia Tech Research Institute. The author is currently with STA Technologies, 5401 Venice Ave, Albuquerque, NM 87113

Abstract

This paper is based on the assumption that the Shroud of Turin is of 14th century origin consistent with its radiocarbon date and historical record and thus must be explained within the technology, social and cultural and contexts of that era. Avoiding the attendant controversy surrounding the date, we present a reasonable plausibility argument to reconcile its visual and forensic properties with extent 14th century printing technology and other related circumstances. We show that striking parallels exist between the general characteristics of an environmentally degraded woodprint and the Shroud image including the pseudo 3-d properties which arise as a natural and unintentional result of the printing process. Existing examples are shown of large woodprints of that era with sufficient detail and contrast variations that resemble the Shroud image resolution and contrast variation. Further the argument is reinforced with analytical results showing that under any reasonable assumptions about the surface bi-directional reflectance distribution function (BRDF) the observed 3-d properties cannot be reconciled with any known radiative imaging process.

Introduction and Background

The Shroud of Turin [1] is a large piece (4.3 x 1.1 m) of complex weave linen containing the faint full frontal and dorsal sepia toned images of a bearded man in repose which appears to be a pictorial representation of the crucified Christ. The image, shown in Figure 1, is replete with detailed wounds and other features revealed by high contrast which are consistent with the gospel accounts of the crucifixion. The authenticated historical record of the cloth dates to circa 1350 A.D. to the Champagne-Ardenne region of France. Many arguments have been presented that the Shroud has a much earlier origins but thus far these conjectures have escaped formal historical authentication. Although art history is rich in representations of the crucified Christ, this particular image has a number unusual attributes which in aggregate have thus far escaped any technological explanation. The origins of the Shroud are highly controversial as reflected in voluminous literature, TV programs and popular non-fiction works[2]. Many regard it as the actual burial cloth of Christ. As appealing as this possibility is, the preponderance of evidence argues otherwise. The Shroud has been radiocarbon dated at the 95% confidence level to between 1260 and 1390 AD [3]in substantial conformity to the Shroud's authenticated historical date of *circa* 1356 AD. There is some controversy about this date in that some of the adjacent fibers were found to contain cotton from an early repair which engendered a conjecture that the radiocarbon thread might have been contaminated thus skewing the measured date forward since cotton is of much later origins than linen.

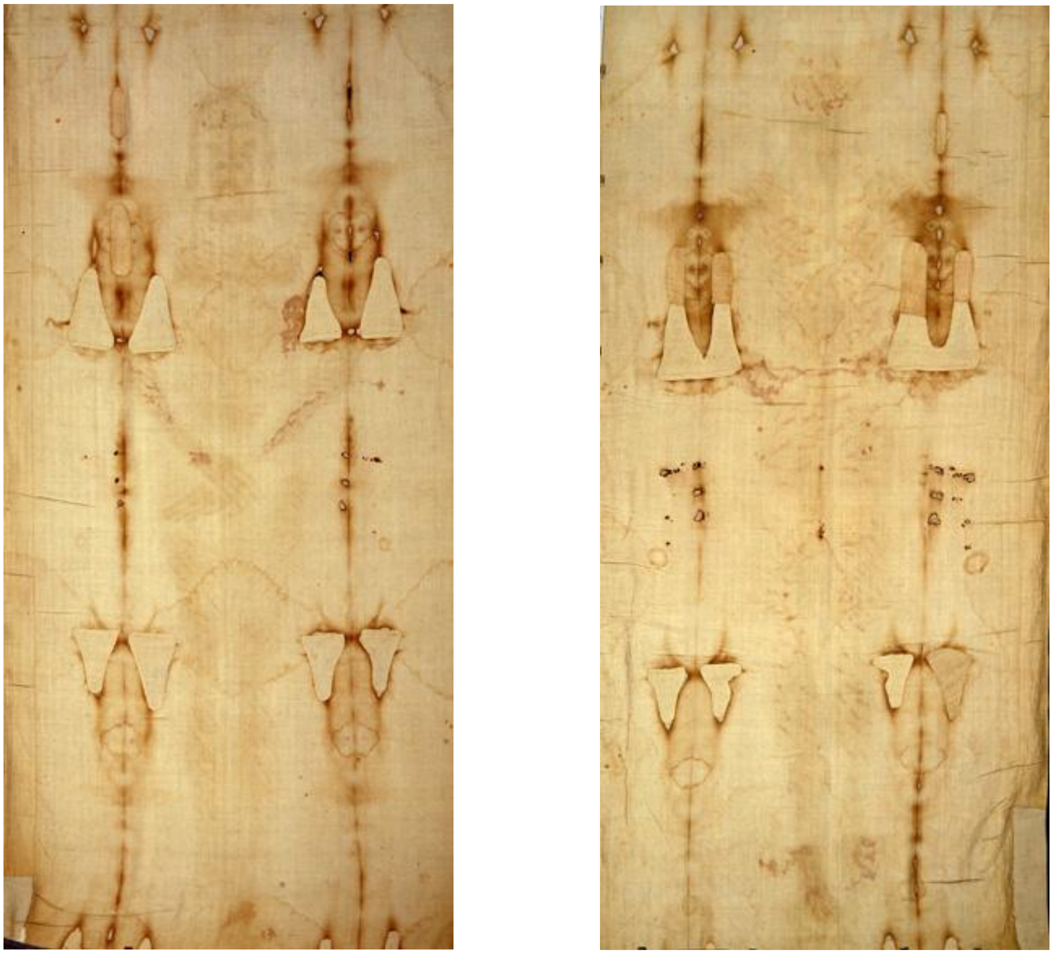

Figure 1. Frontal (L) and dorsal images (R) of The Shroud of Turin

However, it should be asserted that there is no evidence that the author is aware of that the actual fiber that was dated contained cotton. Some work is allegedly in progress to further explore this conjecture and an additional thread known to be from a pristine area of the cloth may ultimately be dated to resolve it. Another conjecture is that the combustion products from the fire of 1532 may have skewed the date as well. That the alleged contamination would somehow yield a radiocarbon date that is consistent with the historical date is a coincidence so profound that it borders on the miraculous. Nevertheless, these contentions, even if true, do not critically impact the argument presented here.

It should be recognized that there could be a difference between the dates of linen production and the image imprinting process in which the former could predate the latter by any number of years. Further, the origins of the linen and/or the image are actually indeterminate. It is clear however that the image and the linen coexisted in the 14^{th} century. A reasonable working hypothesis adopted for the purposes of this paper and justified below is that the Shroud *image* is in fact of circa 14^{th} century European origins.

Image Attributes

There are a number of unusual attributes of this image summarized below.

1) It appears to a singular work with no known parallels of human images in this particular form and medium on very large cloth or tapestries. Images of religious figures are relatively abundant but are usually much smaller in size.

2) The artistic illusion of depth is normally created by perspective or contrast differences. In a full frontal view of the face, the illusion of depth is created by the latter. Under normal lighting, convex features are brighter and the concave features are darker for example in the case of the eye cavities. One of the pathological features of the Shroud image is that these normal contrast expectations are reversed. This is referred to as the *photographic negative* property.

3) It is noted that the normal artistic interpretations are lacking, the aspect angle of the image is parallel to the plane of bilateral symmetry and the image contrast is bilaterally uniform and normal to the plane of bilateral symmetry. No artistic portrayals of illumination direction by way of shadowing are apparent.

4) The image is stark and Gothic in appearance which we note is characteristic of a certain French medieval era in art and boasting a large number of medieval Gothic artists in particular.

5) The image appears to be devoid of normal pigments and brush strokes. Image contrast appears to be entirely attributed to -stain loosely bound to the outmost fibers of the cloth and in a manner similar to a halftone printing whereas darker features contain a higher density of colored fibrils.

6) The image exhibits a rather high resolution estimated at 0.1-0.5 cm . This attribute, in and of itself, suggests a contact image formation process.

7) When observed in isometric projection with an instrument such as the VP-8 Image Analyzer, the image displays a *3-d like property* characteristics of a death mask. By virtue of exploring this particular attribute much light has been shed on the other attribute as discussed below.

The 3-d Nature of the Image

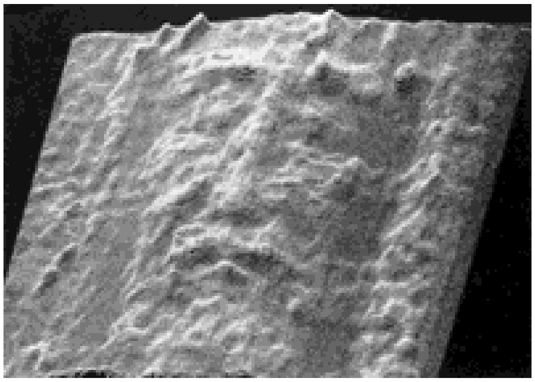

Figure 2. 3-d Relief of the Face as projected by the VP-8 Image Analyzer

The so-called 3-d property of the image arises from an isometric projection of the Shroud image through a VP-8 image analyzer or equivalent. In these instruments, the two dimensional image density function at any point is encoded, projected into the third dimension and displayed isometrically. More specifically, if the 3-d function describing an object is say $z = f(x,y)$ with respect to some arbitrary image plane then a linear function describing the image density p can be assumed to be $p = Kz$ where K is some arbitrary constant. Jackson[4] et.al. analyzed the Shroud with a VP-8 mage analyzer and produced the image shown in Figure 2. There appears to be a relationship between the image point density and the corresponding object point distance yielding the appearance of a crude ”death mask” similar to that shown in Figure 3. There also appears to be a good deal of mottling in the image which can be reasonably interpreted as a non-uniformity or noise in the imprinting process. Much has been made of this so-called 3-d property by the Shroud research community because most images do not display this property.

Although it would be tempting to speculate that an actual death mask as in Figure 3 with spatially invariant bi-directional reflectance distribution (BRDF) photographed with linear film and under direct illumination normal to the object plane would yield a high fidelity 3-d re-creation, as we will show this is not the case. Analogously, the same statement applies to the features of a human face. Firstly, other than passing a subjective plausibility test, the 3-d projection in Figure 3 is rather crude, suggesting the presence of imprinting “noise”. In any case, the actual fidelity of the Shroud image is indeterminate. In human faces there is a relatively wide distribution in the dimensions of plausible facial features and mottling would not be observed. A further complication is that the linearity of the film and hence the print linearity that was subjected to the analysis is unknown. Generally speaking, high contrast film tends to be non-linear with respect to exposure level i.e. the film gamma is non-linear thus some caution is warranted when asserting absolute linearity. It is conceivable that the non-linear characteristics of the film and the actual image may render an object that appears in 3-d much more realistic that it actually is. Further by a process of digital filtering this data an array of esthetically pleasing images can be produced.

Assuming for the moment that such 3-d reproductions are possible, there are several requirements for such a process to render a true reproduction. The object must be devoid of shadowing which inevitably leads to distortion in a 3-d isometric reconstruction. To avoid shadowing, the light source must be directed normally to the object plane and be reasonably isotropic which we speculate is probably difficult to produce with 14th century technology. Furthermore, the hypothetical artist must somehow create a quasi monochromatic portraiture image with uniform density not only with reasonable 2-d fidelity but with reversed contrast 3-d fidelity while interposing himself between the light source and the object and without the appearance of brush strokes. We submit that such a deliberately contrived image would not be esthetically pleasing and is likely outside of the conceptual realm of a 14th century artist. The creation of an image with these properties imposes certain practical difficulties that make this mode of production highly unlikely. Not only is this process *prima facie* implausible because of the requirement to deliberately encode a 3-d effect that would have been totally unimaginable in the presumed time frame but we will show in the appendix that it violates certain radiometric processes. We can only conclude that the result, although remarkable, was an unintentional by-product of the imprinting process.

Figure 3, Death Mask

Although there are a number of 3-d imaging techniques potentially leading to an image contrast proportional to feature distance, these are largely 20th century developments and later. Stereoscopy, laser profilimetry or LADAR, structured light, moiré interferometry are but a few known processes capable of producing a high fidelity isometric image and they are generally object reflectance independent as they must be to avoid deleterious effects.

Image transfer from an object onto some recording medium can only occur by 3 processes; emitted radiation from the object, reflected radiation by the object and direct contact. Since there does not appear to be a radiative process vis-a-vis a lens, human eye or pinhole camera or other device with spatially invariant response that would give rise to the observed 3-d effect, we must specifically look to a direct contact process that renders an image density at least modestly proportional to the object feature depth. For several reasons, high fidelity image transfer in the case of a cloth wrapping a human body would not be possible since the distortion evident upon unfolding would be significant and a good deal of smearing due to front-to-back weight asymmetries would be evident. Further, the image density would be expected to be highly non-uniform which would not render the observed a 3-d effect.

Woodprinting

The only plausible process known to be in use in Europe and more specifically in France during the 14th century that would be consistent with the historical record and radiocarbon date rendering yielding a image with the requisite properties is a contact print known as woodprint or woodcut[5] where the image transfer resembles the halftone density printing process apparent in the Shroud as noted by Rodgers[6]. That the depth of the shallow relief linearly scales to the object dimensions is a reasonable assumption however this relationship may ultimately be relegated to the craft of the artist. Nevertheless, we could reasonably expect that the relief transfers coloring matter in proportional to its local curvature and pressure. Assuming the average pressure over the entire block is reasonably constant, local relief curvature would dominate the density of the image transfer process. Such a process would also produce the "photographic negative" property of the image in the Shroud where the normal expectations of contrast between convex and concave features of the face are reversed. On the basis of the existence of many fine printed tapestries with exquisite detail we can infer that this process is clearly able to reproduce the resolution observed in the image. Examples of sculpture and engravings during this period demonstrate sufficient skill for the image details observed although there are no references to full scale printed human images. Because of the engraving skill required it is quite likely that such a woodprint would have been created by an engraver however it is also noted that woodprints were frequently done in a two stage process involving first an artist/designer and then an engraver.

Gothic art[7] emerged in France in the mid 12th century in sculpture of religious objects. A review of 14th century French art reveals little in the way of classical brush painting before this time but much in the way of woodcuts and intaglio both of which were imported from the far east and used to print fabrics and then later on paper. The following historical synopsis is taken from the Encyclopedia Britannica [8].

> *Woodcuts are a technique of printing designs from planks of wood incised parallel to the vertical axis of the wood's grain. It is one of the oldest methods of making prints from a relief surface, having been used in China to decorate textiles since the 5th century ad. In Europe, printing from wood blocks on textiles was known from the early 14th century, but it had little development until paper began to be manufactured in France and Germany at the end of the 14th century. Cuts with heavy outline and little shading, as the "Christ Before Herod" (British Museum), may date from 1400, while the earliest dated print of German origin is the Buxheim St. Christopher of 1423. In Bavaria, Austria, and Bohemia, religious images and playing cards were first made from wood blocks in the early 15th century, and the development of printing from movable type led to widespread use of woodcut illustrations in the Netherlands and in Italy. With the 16th century, black-line woodcut reached its greatest perfection with Albrecht Dürer and his followers Lucas Cranach and Hans Holbein. In the Netherlands Lucas van Leyden and in Italy Jacopo de' Barbari and Domenico Campagnola, who were, like Dürer, engravers on copper, also made woodcuts.*

We note from the above that printing on textiles with woodblocks was in practice in the early 14th century and these early prints were generally of religious origins. There are not many surviving works on textiles from this era and that the French woodcut images that do survive seem to be rather crude. Although it would appear that the size of such a woodblock would be formidable, several woodblocks were used to create a large print apparently having solved the problem of registration and uniformity in the production of high quality prints. Tapestries or printed wall hangings with religious subject material from this period were very common and noted as being "very imposing" in size. The image is stark and Gothic in appearance [9] which we note is characteristic of a certain French medieval era in art and boasting a large number of medieval Gothic artists in particular. Chamberlin[10] cites that illustrations from this period were typically and unmistakably Gothic and that textile printing preceded books. One print is described as being 4 x 9 ft in size together with its 6 engraved associated woodblocks. Two other large creations of note are on display in the Palace of the Doges Museum including d' Barberi's *Map of Venice* shown in Fig.4 and the large blocks used to create a world map in Fig.*5.* It is clear from the foregoing that there are ample historical precedents for printing very large creations the size of the Shroud.

Fig.4 Woodprint of Map of Venice

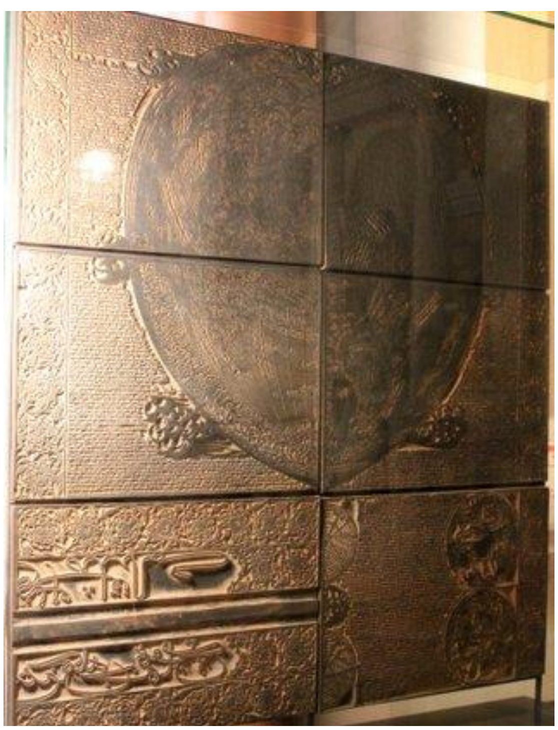

Fig.5 Woodblock engraving of world map

Although there seems to be no evidence of the type of linen weave observed in the Shroud being manufactured in France during the 14th century, Burke [11] noted that the survivors of the black death in 1347 became relatively wealthy from inheritances and that created a great demand for luxury goods from the Far East including linen. Thus linen became commonly available and served as raw material for the nascent paper industry which ultimately led to large scale book production. The Champagne region of France at this time was a noted trade center and so it is also a reasonable proposition that the linen may not have been of local origins although there was a thriving linen and textile printing industry in Flanders from the 11th century on.

A review of the history of textile printing[12] reveals the following:

> *"Textile printing was known in Europe, via the Islamic world, from about the 12th century, and widely used. However the European dyes tended to run, which restricted the use of printed patterns. Fairly large and ambitious designs were printed for decorative purposes such as wall-hangings and lectern-cloths, where this was less of a problem as they did not need washing. When paper became common, the technology was rapidly used on that for woodcut prints. Superior cloth was also imported from Islamic countries, but this was much more expensive*."

The last sentence underscoring Burke's comments regarding the importation of expensive cloth. From the same reference, it appears printing of cotton was well developed in medieval France. The article goes on:

> *"From an artistic point of view most of the pioneer work in calico (crude cotton or muslin) printing was done by the French; and so rapid was their advance in this branch of the business that they soon came to be acknowledged as its leading exponents. Their styles of design and schemes of colour were closely followed-even deliberately copied by all other European printers; arid, from the early days of the industry down to the latter half of the 10th century (sic), the productions of the French printers in Jouy, Beauvais, Rouen, Alsace-Lorraine, &c., were looked upon as representing all that was best in artistic calico printing. This reputation was established by the superiority of their earlier work, which, whatever else it may have lacked, possessed in a high degree the two main qualities essential to all good decorative work, viz., appropriateness of pattern and excellent workmanship."*

We would speculate that it would be somewhat easier to engrave the observed detail on a larger image. We have made note of the fact that the frontal and dorsal images are reasonably uniform and consistent with each other in image density. The participation of a human body in this process would have dictated otherwise. Clearly, it would not be particularly challenging to engrave a woodblock of this size considering that some of the sculptures that exist from this time frame are quite detailed and full sized. Exquisite 5th century Roman carvings fare on display in Venice thus it appears that woodcarving was a well-developed art form by the 14th century. Although somewhat pedestrian in regard to the much more exotic image formation processes that have been conjectured, a woodprint is a plausible, *the most plausible* process extant in the early 14th century and that leads us to the question of the ink or coloring agent that accounts for the image.

Coloring Agents

Initially the colorant was thought to be degraded cellulose and Jackson [13] explored a candidate process using a hot statue to induce a color change by virtue of thermal cellulosic degradation. The author speculated about the woodprint process and iron gallate inks nearly 30 years ago but it was not given serious attention at the time because of this finding. Although iron gallate inks were prevalent during this time frame and would have led to cellulosic degradation, no chemical evidence supporting this conjecture was found. Subsequently, Rodgers [14] found that cellulosic degradation was not the image colorant and that the image density resembled a half-tone printing process where the contrast is a function of local colorant density[15].

Adler found that the coloring agent persisted on only the very top fibers and could be removed by a diimide reaction[16]. Rodgers also observed that there is no diffusion of the coloring into the threads. Aldehyde and carboxyl groups were identified as present in the image areas of the cloth. Further that the agent could be physically peeled off from the fibrils displaying a pristine uncolored fibers underneath leading to the conclusion that the cellulose did not participate in the coloring reaction dispelling any existing notions about the coloring agent being a chemical or thermal scorch.

The missing link then as it is now is the connection between what is known about the surface chemistry of the fibrils and potential chemical properties and effects of the various inks or dyes that might have been used in the 14th century. To pursue an explanation that is consistent with this observation we have to look to a viscous coloring agent or dye that would be compatible with the printing process by being spreadable with reasonably uniform consistency over the relief. As is widely known, linen is difficult to dye and consistent with the known Shroud repair with cotton fiber then subsequently dyed to match the yellowing of the linen such as has been observed in the Shroud repair. However, to faithfully reproduce the gospel accounts, the use of linen as the backdrop for the image was essential else authenticity would have been in doubt. A large linen and textile printing industry was in existence in nearby Flanders in this time frame. Linen weaving was a cottage industry thus knowledge of the manufacture and dyeing of linen and of textile printing was locally practiced and thus reasonable that both the linen and technology to print the image easily migrated to Lirey.

An agent which did not directly bind with the cellulose would have gradually washed away over time and explains the very faint quality of the current image; too faint for an artist to evaluate his skills during creation. Clearly the artist would have had to have a prominent image to insure that his skills were properly displayed and further that the image display had sufficient detail to be convincing. There seems to be little point in the inscription of the immense amount of detail without adequate visibility to the naked eye. Iron gallate and carbon inks of which there are reportedly several hundred recipes are seemingly plausible candidates for coloring agents because they were the ink of choice during the medieval period and could be made sufficiently viscous with gum Arabic, linseed oil or a number other binders to serve as suitable colorant for textiles. It should be noted that the penetration of the colorant into the textile was not a primary consideration during the printing process because mechanical stresses would have flaked almost all of the ink away. Many of these formulations are corrosive and would have over time either caused cellulosic degradation or reacted with some other substance on the linen such as a mordant or starch. Some of these inks are known to create time dependent contrast effects depending on the formulation and there are several hundred papers dealing with the conservation issues on rare documents relative to these inks [17]. Although given the number of possible recipes and the contaminants that may have existed on the Shroud prior to imprinting it is not possible to rule these out as candidates. We do note suggestively that an abnormal amount of trace iron was found uniformly embedded in the fabric of the cloth [18]. We could speculate that the starch likely present in the cloth reacted with the gallate ink rather then the cellulose itself. Gum Arabic which is a complex and variable mixture of arabinogalactan oligosaccharides, polysaccharides and glycoproteins and a common binder for coloring agents and may have been responsible for the effect observed by Rodgers and Adler that the sepia coloring agent literally peeled off from the fiber leaving only the uncolored underlying fiber however no protein in the image areas was found [19]. The initial protein may be bound chemically in ways not amenable to test. In summary, there is a relatively wide range of chemical reactions that that have not been investigated and further the colored fibrils should be subjected to all of the means of modern analytical techniques to specifically define the molecular composition of the coloring and this will require attendant access to the cloth for further sampling.

The following text is revealed insofar as it describes the difficulty of dyeing linen and was summarized from reference [20]. Because linen is inherently difficult to dye, it must be prepared with a mordant before attempting any pigmentation or coloring. The linen is pretreated with sodium sulfate or ammonia or even stale urine. The chemicals are added to boiling water and the linen soaked until the fibrils are softened. It is the adjustment of pH within natural fibers that allow the dissolved pigments in natural dyestuffs to enhance and color fabrics. Further the mordants do affect the final color. Once the pH of pH9-pH10 has been adjusted and the fabric conditioned, it can be rinsed and mordanted with alum or a combination of alum and tannic acid. There are many different techniques for this process and many are based on a percentage of dry fabric weight to compound or either water volume per container**.** An acceptable pH range for dyeing linen is around pH7.5-pH8

The most telling revelation in this article is:

> *"Due to the structural binding of the cellulose fibers in linen, the pigments will only adhere to the surface of the fibers. While linen will dye just as vibrant and deeply as other fibers, it will not retain its color as long. Exposure to air, light and chemicals speed the deterioration process."*

It is highly suggestive from the preceding passage that the original image probably faded a good deal over time. It is not clear if the artist necessarily pretreated the linen with mordent however it would have rendered a greater image visibility and a longer lasting imprint. It is also clear that the washing of the Shroud from time to time had a deleterious effect on the image contrast.

Summary and Conclusions

In summary, we have presented a reasonable plausibility argument that the Shroud image must result from a contact process. Woodblock or intaglio techniques known to be in use in 14th century in Europe and in France account for all of the visible attributes of the Shroud image including the 3-d effect, reversed contrast, the resolution, uniformity between the frontal and dorsal images and the extensive detail observed. The confluence of a number of historical and circumstantial factors in 14th century France including local textile linen production and printing techniques, the emergence of the Gothic art form and the peculiar circumstances surrounding the black death underscore the woodprint nature of the Shroud. The nature and longevity of dyed or colored linen is consistent with the observation that the colorant lies on the very top surface of the fibrils and that the cellulose did not participate in the reaction. The ephemeral nature and difficulty of dyeing linen speaks to the very faint image currently observed on the cloth. The exact chemistry of the coloring agent is unknown but may result from several hundred potential recipes for ink or coloring matter and binders reacting with mordant preparation and starch impurities in the cloth and binders such as gum Arabic or linseed oil. The pervasive presence of iron is highly suggestive of iron gall ink. Further investigations on the specific chemical nature of the colored fibrils are necessary to identify which of these processes may have been used. Lastly we have shown that it is quite unlikely that the image resulted from any radiative process not to mention the fact that this process would transcend any known physics. In the authors opinion these are relatively compelling reasons to assert that the 14th century Shroud of Turin image is the remnant of a woodprint or similar intaglio. It may represent an unusually high quality print for its era.

References

1. Wilson, I, *The Shroud of Turin*, rev.ed (Doubleday, 1979)

2. For a bibliography and guide to the numerous publications see The Shroud of Turin Website, http://www.shroud.com/

3. P. E. Damon, D. J. Donahue, B. H. Gore, A. L. Hatheway, A. J. T. Jull, T. W. Linick, P. J. Sercel, L. J. Toolin, C.R. Bronk, E. T. Hall, R. E. M. Hedges, R. Housley, I. A. Law, C. Perry, G. Bonani, S. Trumbore, W. Woelfli, J. C. Ambers, S. G. E. Bowman, M. N. Leese & M. S. Tite, Radiocarbon dating of the Shroud of Turin, *Nature*, Vol. 337, No. 6208, pp. 611-615, (February, 1989)

4. Jackson, J.P.: Jumper, E.J. & Ercoline, W.R.: Three-dimensional characteristics of the Shroud Image. In: Proceedings of 1982 IEEE Int. Conference of Cybernetics and Society, p. 559-575

5. A. R. Hind, *An Introduction to a History of Woodcut* (Dover 1963)

6. R. N Rodgers, A Chemists Perspective on the Shroud of Turin, Barrie Schwortz Pub. 1982 The Shroud of Turin Website, http://www.shroud.com

7. See for example :Emile Male, *The Gothic Image , Religious Art in France of the Thirteen Century*, p 165-8, English trans of 3rd ed. 1913, Collins, London (and many other editions)

8. "woodcut." Encyclopædia Britannica. 2008. Encyclopædia Britannica Online. 11 Dec. 2008 <http://search.eb.com.proxygsu-git1.galileo.usg.edu/eb/article-9077414>.

9. Related to the author at a gathering of the art faculty of the University of New Mexico (personal communication, 1979)

10. W. Chamberlin, *Woodcut Printmaking*, (Hermes & Hudson,1978)

11. G. Burke, J. *Connections*, Little- Brown Boston, Toronto, 1978 pp100-101

12. "printmaking." Encyclopædia Britannica. 2008. Encyclopædia Britannica Online. 11 Dec. 2008 <http://search.eb.com.proxygsu-git1.galileo.usg.edu/eb/article-28329>.

13. J. P. Jackson, E. J. Jumper, W. R. Ercoline, "Correlation of image intensity on the Turin Shroud with the 3-D structure of a human body shape," Appl. Opt. **23**, 2244-2270 (1984).

14. L. A. Schwalbe, R. N. Rodgers, Physics and Chemistry of the Shroud of Turin, Analytica Chemica Acta, V 135, pp 3-49, 1982

15. ibid p 34

16. J. H. Heller and A. D. Adler, A Chemical Investigation of the Shroud of Turin, Canadian Society of Forensic Science Journal, 14 (1981), pp 81-103

17. The Ink Corrosion Website, http://www.knaw.nl/ecpa/ink/

18. V. D. Miller and S. F. Pellicori, "Ultraviolet fluorescence photography of the Shroud of Turin", Journal of Biological Photography, **49**, 71-85 (1981).

19. TBD

20. Lady Gwenhwyvar ingen Greig, *Techniques for Pre Treating & Mordanting Linen* http://www.housedragonor.org/A&S/Mordanting.html

21. F. E. Nicodemus, *Reflectance nomenclature and directional reflectance and*

emissivity, Appl. Optics, 9 (6), 1474–1475. (1970)

22. B. T. Phong, Illumination for computer generated pictures, Communications of ACM 18 (1975), no. 6, 311–317.

23. S.R. Marschner, S. H. Westin, E. P. F. Lafortune, K. E. Torrance, and D. P. Greenberg Image-based BRDF Measurement Including Human Skin. In Proceedings of *10th Eurographics Workshop on Rendering*, pages 139-152. Granada, Spain, (June 1999).

Appendix
Analysis

The analysis presented herein seeks to show that given some reasonable assumptions of the reflectance distribution function of the objects surface including that of human skin, the observed 3-d effects are not explicable in ordinary radiometric imaging processes including that of the human eye. We choose a feature function that represents the approximate dimensions of the human nose as a test case and compare the point by point radiance of this image as projected into the third dimension by a "VP-8 like" instrument for fidelity against the actual shape of the test object.

As is well known, surfaces do not radiate or reflect radiation isotropically. The spatial distribution of radiation reflected or emitted from a surface is a function of material, surface properties and geometry between the light source, object and observer and is conveniently

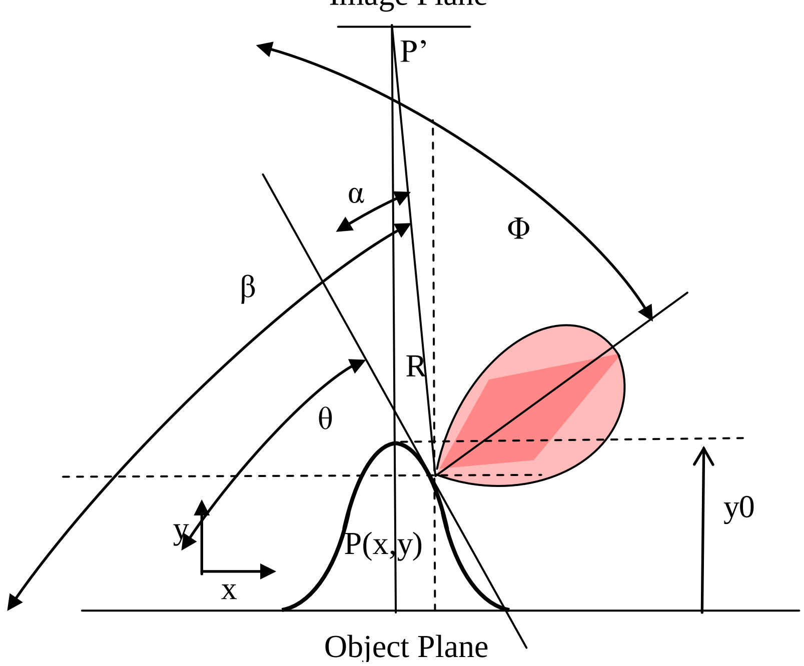

Figure 6. Feature function and the BRDF distribution

described by the bidirectional reflectance distribution function BRDF [21]. Of interest is the relative radiance or other radiometric quantity at point P' in the image plane as a function of the position of the conjugate object point P and the form of the BRDF function. Referring to Fig.6, we define an approximate analytic function to model the two dimensional profile of the human nose at its extremities as:

$$P = f(x, y) = y_0 \cos^2(bx) \tag{1}$$

The illumination direction is **–y** or normal incidence**.** The angle of interest which define the relative radiance in the image point direction are:

$$\Phi = \frac{\pi}{2} - \alpha\,, \tag{2}$$

$$\beta = \theta + \alpha\,, \tag{3}$$

$$\beta = \frac{\pi}{2} - \tan^{-1}\left(\frac{x}{R-y}\right), \tag{4}$$

and $$\theta = \tan^{-1}\left(\frac{dy}{dx}\right) = \tan^{-1}(-2by_0\cos(bx)\sin(bx)) \tag{5}$$

Thus $$\Phi = \frac{\pi}{2} + \tan^{-1}\left(\frac{x}{R-y}\right) + \tan^{-1}(-2by_0\cos(bx)\sin(bx)) \tag{6}$$

We define a relative parametric Phong [22] BRDF function as:

$$\cos^n(\Phi) \text{ where } \Phi = g(x,y) \tag{7}$$

and n is a parameter that will be varied to simulated various forms of the BRDF and the result compared to the normalized feature function.

We note qualitatively that at increasing x, Φ the angle between the line of bilateral symmetry of the BRDF and the y axis increases, exposing less and less radiation to the image point *P'*. In addition there is a $1/R^2$ effect as the distance to *P* denoted by *R* becomes slightly larger as we move down the feature. As the object point approaches the object plane the $cos^2(ax)$ feature function flattens out again and Φ decreases. At $R >> y_0$, the $1/R^2$ effect is almost negligible so in general we expect to see the radiance at point *P'* first decrease then increase as a function of x. Quantitatively that is exactly what happens. Fig. 5 below shows the normalized radiance as a function of several assumed Phong approximations $cos^n(\Phi)$ to the BRDF for R= 1000 and R= 50 units. The feature function height ~ 22.0 mm and the feature function half-width ~ 15.0 mm resembling the approximate proportions of the author's nose. The data is presented in normalized form. Of particular note is the result from the measured BRDF [23] of human skin also shown in Fig. 6. In no instance does the normalized radiance function resemble the geometrical feature function requisite condition for a realistic 3-d reconstruction. Note that at the extremities of the width function relative the radiance approaches its value at 0. This condition would essentially translate to the cheekbones being at the same height as the tip of the nose in isometric projection as there is negligible radiance difference between the two points. Without asserting the existence of pathological conditions totally uncharacteristic of normal objects such as a carefully defined spatially varying hemispherical reflectance, BRDF or absorptive media, the result does not support a radiometric reflective or emissive image formation process.

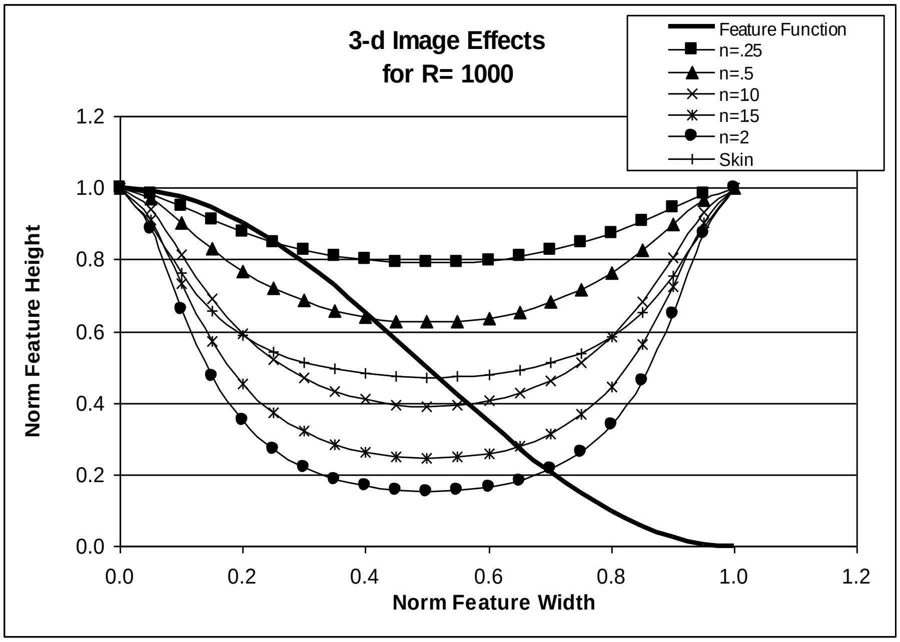

Figure 7. Normalized radiance function compared to the assumed feature function at R=1000 for various Phong $\cos^n(\Phi)$ BRDF functions

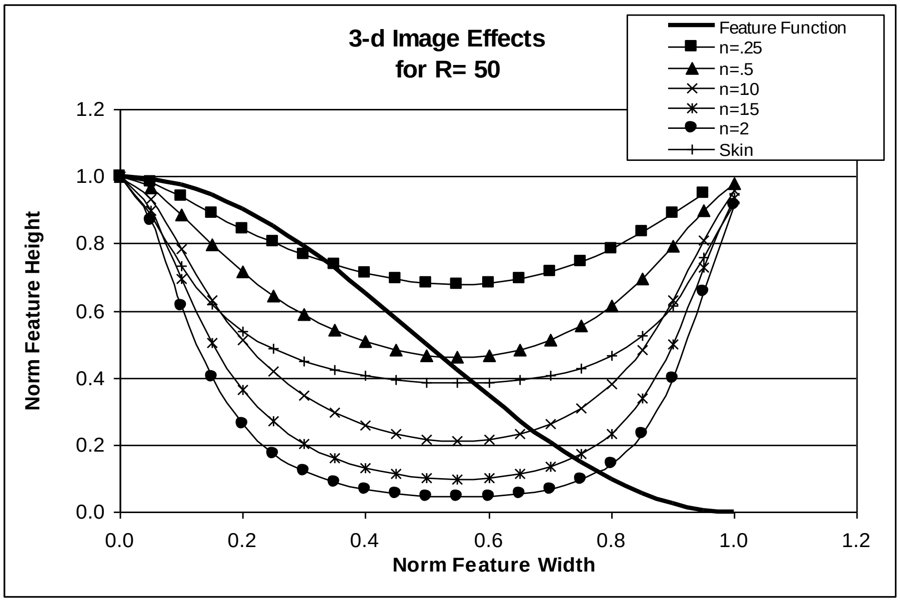

Figure 8. Normalized radiance function compared to the assumed feature function at R=50 for various Phong $\cos^n(\Phi)$ BRDF functions

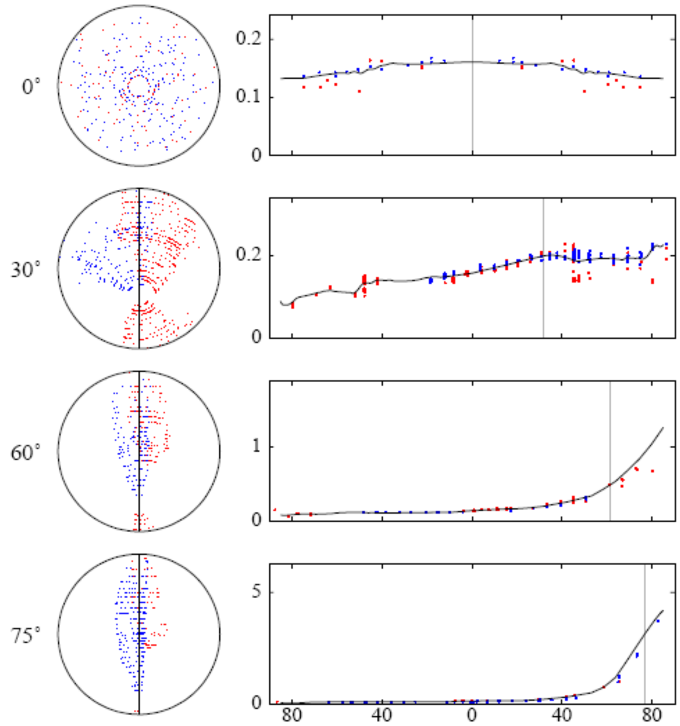

Figure 7. BRDF measurement of human skin taken from reference 9. The 0 degree case in the one of interest for the assumed normal illumination

Thus, using any one of a number of Phong $\cos^n(\Phi)$ approximations to the BRDF distribution including the measured BRDF of human skin, realistic 3-d features cannot be reproduced in isometric projection. This result appears to hold true under a number of conditions including a wide range of image distances, two of which are shown above and an illumination source that is normal to the object plane to avoid the shadow distortion. Thus we must look to another 14th century (or earlier) process to explain the apparent 3-d character of the image including the resolution, reversed contrast, lack of pigments and lack of artistic license.